

A METHOD FOR THE ANALYSIS OF ARTIFICIAL CLOUDS IN THE UPPER ATMOSPHERE

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ABSTRACT

A new method of analyzing visible artificial cloud in the upper atmosphere is described by which we are able to select correctly any corresponding sections of a cloud from a pair of observation films.

Applying this method to several rocket experiments, rocket trajectories and wind velocities at various altitudes were deduced. It is concluded that the magnitudes of errors in this method are about 0.1-1.0 km in position and about several meters per second in velocities.

1. Introduction

Artificial clouds produced by rocket experiments have proved to be a useful tool in the study of winds and other phenomena in the upper atmosphere.

In this paper, we describe a new method for the analysis of artificial clouds. Some analytical methods have already been developed by various authors such as Broglio (1961), Justus *et al.* (1964). But all of them have encountered difficulties in identifying any corresponding points of a cloud in a pair of observing films, as an artificial cloud has finite dimension.

In our method an attempt was made to obtain several cross-sections of the cloud by using the line between a pair of observational stations as the base line for projecting a set of planes at different elevations and consequently we were able to select any corresponding sections of a cloud in a pair of film records obtained at the two stations (Nakamura *et al.*, 1968).

2. Outlines of Analytical Method

Let P_A be the unit vector directed from station A to the cloud, γ_A be the distance between station A and the cloud, and Q_A be the position vector of station A with the origin at the center of the earth. Then the position vector C_A of the cloud is denoted by

$$C_A = Q_A + \gamma_A P_A, \quad (1)$$

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where Q_A and P_A are known quantities and γ_A is unknown.

As to the other station B, the position vector C_B of the cloud is expressed by

$$C_B = Q_B + \gamma_B C_B. \quad (2)$$

From the condition

$$C_A = C_B, \quad (3)$$

we shall determine γ_A and γ_B . But this relation (3) does not hold exactly due to errors of many kinds. So γ_A and γ_B will be determined by the least square method, that is $|C_A - C_B|^2$ has minimum value.

Thus

$$\left. \begin{aligned} \gamma_A &= (Q_A - Q_B) \cdot [(P_A \cdot P_B) P_B - P_A] / \{1 - (P_A \cdot P_B)^2\}, \\ \gamma_B &= (Q_B - Q_A) \cdot [(P_A \cdot P_B) P_A - P_B] / \{1 - (P_A \cdot P_B)^2\}. \end{aligned} \right\} \quad (4)$$

Then the most probable position vector of the cloud may be expressed as

$$C_{AB} = (\gamma_B C_A + \gamma_A C_B) / (\gamma_A + \gamma_B). \quad (5)$$

Now in the determination of P_A or P_B , we encounter a serious problem, for artificial clouds have finite dimensions, that is, it is difficult to confirm that P_A and P_B really direct towards the same point in the cloud.

Let us consider a plane passing through stations A and B. The projection of this plane on the film obtained at station A or B will be straight line. And the segments of cloud images cut by these lines in each film must be the same part of the cloud. Using this fact, we may proceed with our analysis.

In the geocentric coordinate system with the origin at the center of the earth, three Cartesian coordinate axes were defined as follows;

- (z) directed toward the geographic north pole,
- (x) directed toward the point of intersection of the equator and the meridian which passes through KSC,*
- (y) perpendicular to both x and z axes.

In this system, the position vector of station A is expressed as

$$Q_A = \begin{pmatrix} (\rho(\varphi_A) \cos \varphi_A' + h_A \cos \varphi_A) \cos (\lambda_A - \lambda_1) \\ (\rho(\varphi_A) \cos \varphi_A' + h_A \cos \varphi_A) \sin (\lambda_A - \lambda_1) \\ \rho(\varphi_A) \sin \varphi_A' + h_A \sin \varphi_A \end{pmatrix}, \quad (6)$$

where

- φ_A geographic latitude of station A; φ_A' geocentric latitude of station A;
- λ_A longitude of station A; λ_1 longitude of KSC;

* KSC means Kagoshima Space Center and is the main observational station of our rocket experiment. Its position is $\varphi_1 = 31^\circ 15' N$, $\lambda_1 = 131^\circ 05' E$.

h_A altitude of station A; $\rho(\varphi_A)$ radial distance of a point at sea level whose latitude is φ_A .

Here the earth was considered to be a perfect ellipsoid of revolution having eccentricity of 0.081820.

It is convenient to choose the coordinate system M_A in which the z -axis coincides with the vector directed from station A to B and the origin lies at the location of station A. The x -axis is in a plane determined by the z -axis and the celestial north pole and perpendicular to the z -axis and lies opposite side to the celestial north pole. The y -axis may be determined so as to form right-hand Cartesian coordinate system. Another coordinate system M_B is also defined so that its origin lies at station B and three axes are parallel to M_A . Any part of the cloud will be represented by the angles μ, σ_A in the M_A system or μ, σ_B in the M_B system as shown in Fig. 1, where P_A and P_B are the unit vector from station A and B to the cloud. In the coordinate system M_A and M_B , the plane passing through both stations A and B, is expressed by $\mu = \text{const.}$, which is called μ -plane.

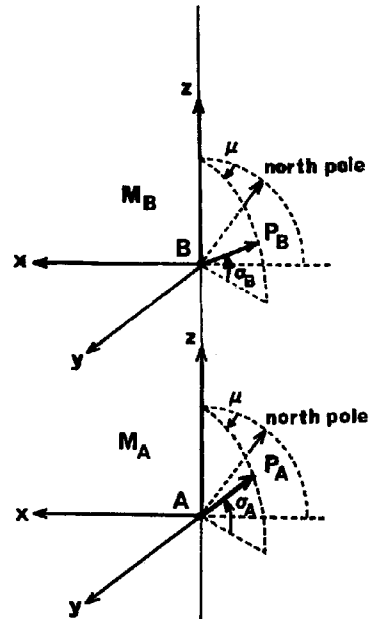


Fig. 1. Definition of the angles μ, σ_A and σ_B in the M_A and M_B system.

Fig. 2 shows an example of μ -plane projections to the films of both stations. In this case the tangential points C and D shall correspond exactly to the same point of the cloud. On the other hand the segment A will correspond to the segment B.

If we read the values of σ corresponding to end points $\sigma_{A1}, \sigma_{A2}, \sigma_{A3}$ and to the middle points σ_{A2}, σ_{B2} , we can designate six unit vectors P_{A1}, P_{A2}, P_{A3} at station A and P_{B1}, P_{B2}, P_{B3} at station B. As P_{Ai} and P_{Bi} lie in the same plane, γ_{Ai}

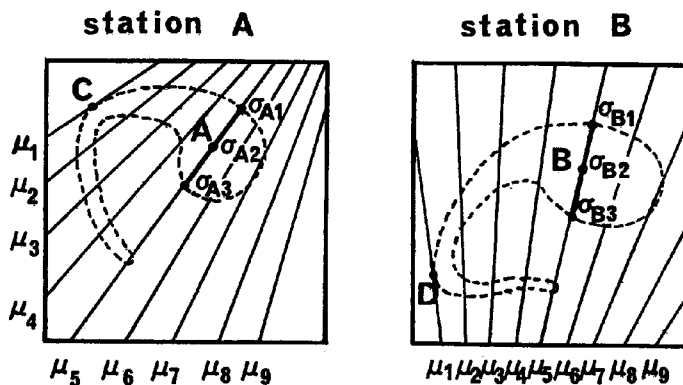


Fig. 2. μ -plane projections to the films of station A and B.

and γ_{Bi} obtained from eq. (4) satisfy the relation (3). Thus the cross-section A, B in this μ -plane can be calculated as a quadrilateral shown in Fig. 3.

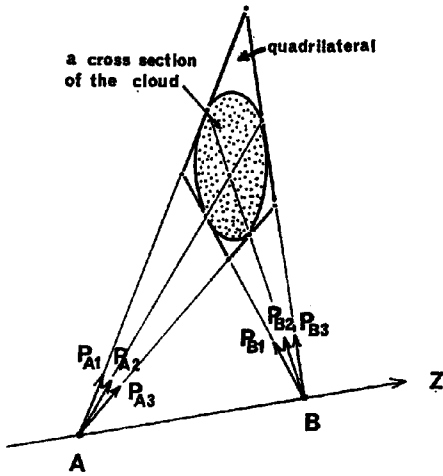


Fig. 3. A cross-section of the cloud in a μ -plane.

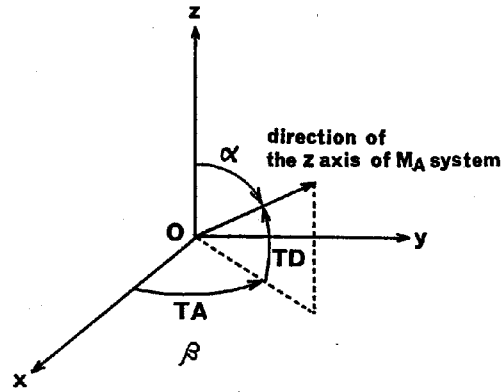


Fig. 4. Definition of angles TA and TD in the geocentric coordinate system.

The unit vector $P_A(\mu, \sigma_A)$ in the M_A system is expressed as

$$P_A = (-\cos \sigma_A \cos \mu, \cos \sigma_A \sin \mu, \sin \sigma_A). \tag{7}$$

Consequently unit vector P_A in the geocentric coordinate system is expressed as follows;

$$P_A = R \times (-\cos \sigma_A \cos \mu, \cos \sigma_A \sin \mu, \sin \sigma_A), \tag{8}$$

$$R = \begin{pmatrix} \cos \beta & -\sin \beta & 0 \\ \sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{pmatrix}, \tag{8'}$$

$\alpha = 90^\circ - TD \qquad \beta = TA$

where TD and TA are the elevation and azimuthal angle of the directions of z-axis of M_A system respectively in the geocentric coordinate system. They are illustrated in Fig. 4. The vector P_B is expressed in a similar way. From equations (4)~(8) we can obtain C_{AB} vector of any part in the clouds in the geocentric coordinate system.

Changing the angle μ , we may obtain other quadrilateral sections in the geocentric system. The final position and shape of the artificial cloud may be determined by synthesizing the results obtained from all the pairs of stations. Thus the wind velocities and directions may be obtained from successive film pairs corresponding to different times.

3. Analysis of Film

The plane coordinate system F in a film is shown in Fig. 5. In this system, the origin is the center of frame, the y -axis is directed toward the celestial north pole in the film, and the x -axis is perpendicular to the y -axis and directed westward in the film. The direction of the camera axis to space is represented by the decli-

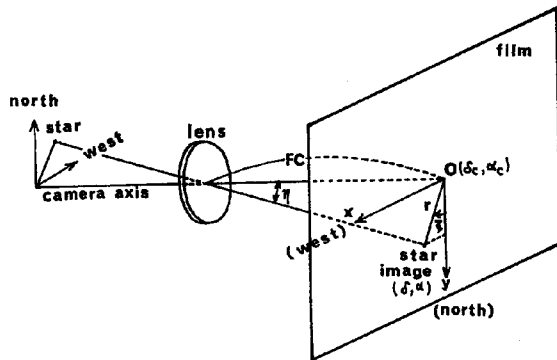


Fig. 5. Relation between plane coordinate system F and a star.

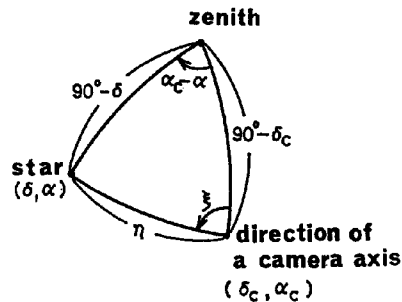


Fig. 6. The spherical triangle with the zenith, the direction of a camera axis and the star, as its vertices.

nation, δ_c , and the right ascension, α_c . The declination, δ , and the right ascension, α , of a star was obtained from the star catalogue of Bečvář (1964) by taking into account the proper motion of a star. The film coordinates of a star are obtained by solving the spherical triangle (Fig. 6);

$$x = r \sin \xi, \quad y = r \cos \xi, \quad (\cos \xi > 0 \text{ when } \delta > \delta_c, \cos \xi < 0 \text{ when } \delta < \delta_c),$$

where

$$\begin{aligned} \sin \xi &= \cos \delta \sin (\alpha_c - \alpha) / \sin \eta & (\sin \eta > 0), \\ \cos \eta &= \sin \delta \sin \delta_c + \cos \delta \cos \delta_c \cos (\alpha_c - \alpha), \end{aligned}$$

provided

$$r = \overline{FC} \tan \eta \quad \text{where } \overline{FC} \text{ is focal length of camera lens.}$$

The values (μ_c, δ_c) of the film center are obtained from the elevation angle and direction of camera axis by the coordinate transformation. Thus we can plot the (μ, σ) -mesh together with stars in the F plane. Project the film image on this F plane using the UP-350 type shadowgraph with magnification of five times, then the images of stars will coincide with plotted ones with precision of about 0.5 mm. Reading σ value of certain μ value section in the cloud, we can calculate the position of the cloud by equations (4)~(8).

Location of five observational stations are shown in Table 1. The longitude and

Table 1. Locations of Observational Stations.

Station No.	Station	Longitude λ °E	Latitude φ °N	Altitude H km
1	KSC	131.08350	31.25317	0.300
2	Nango	131.37414	31.53039	0.014
3	Tanegashima	131.00920	30.72554	0.115
4	Yamagawa	130.61782	31.20036	0.074
5	Taniyama	130.53033	31.52639	0.015

latitude of these stations were remeasured for the purpose of this analysis with an error of within about 10 m horizontally and about 2 m vertically. At each station, K-20 type cameras with lenses having focal ratio 3.5 and focal length of 200 mm were used. The field of a camera was $35^\circ \times 45^\circ$. An exposure was made every 15 seconds or 30 seconds for five seconds, and photographs were taken for about seven minutes. Simultaneous exposures of each station were regulated by quartz chronometer adjusted to within 0.1 second.

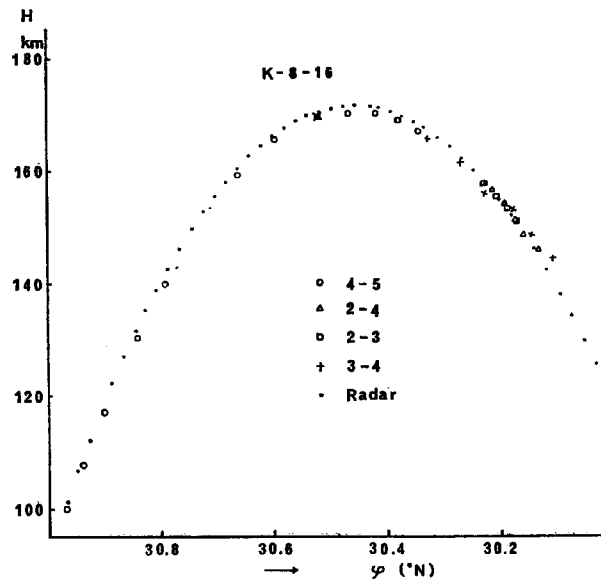
The above analytical method showed a systematic error when the older values of positions or heights of camera stations or α , δ , the value of a star for the epoch 1950.0 were used. However, using corrected values for position or α , δ , these errors were almost removed. This is considered as one piece of the evidence confirming the correctness of our method.

4. Applications and Discussions

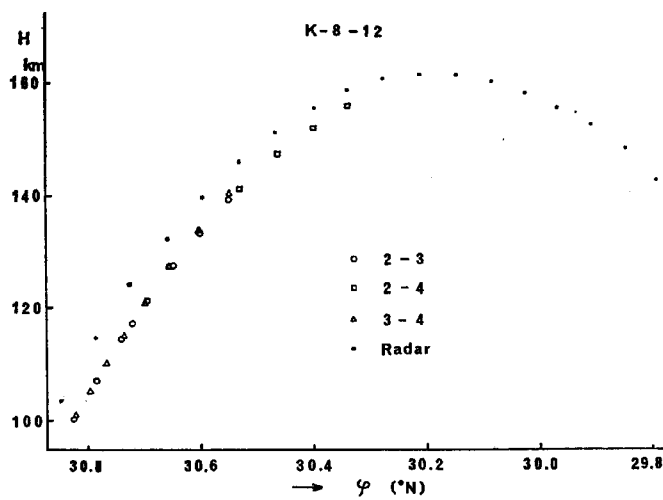
4.1. Determination of the Rocket Trajectory

The K-8-12 rocket was launched from Kagoshima Space Center at 1950 JST (135° EMT) on 16 July 1965 and clear sodium vapour trails were observed. The K-8-16 rocket was launched at 1914 JST on 2 Sep. 1970 and the light from Ba-burning in the rocket was observed. At the same time tracking was also carried out by radar observation.

Applying the above analytical method to these optical trails, trajectories of the K-8-12 and K-8-16 rockets were calculated as shown in Fig. 7 in which the results of a radar observation are also shown. In Fig. 8 the deviations between these two methods are shown. In the K-8-16 rocket experiment, agreement is satisfactory, but in the K-8-12 experiment some systematic discrepancies are noticed. These discrepancies may be due to following cases: (1) In the K-8-12 experiment, a small radar antenna of 2 m diameter was used, while in the K-8-16 experiment it was of 4 m diameter. (2) The position of 2 m radar may not have been precisely determined. (3) Coincidence of the optical axis and the antenna axis of the 2 m parabola may not have been good. (4) Directional characteristics of the 2 m parabola were not sharp.



(a)



(b)

Fig. 7. Rocket trajectories deduced from this new method and radar method. (a) K-8-16, (b) K-8-12. Pairs of numbers correspond to a pair of observational stations shown in table 1.

(5) The sodium cloud at K-8-12 experiment had finite dimensions.

For these reasons, the results from 2 m radar may have contained some errors.

It may be seen that this method will provide a reasonable check for other methods.

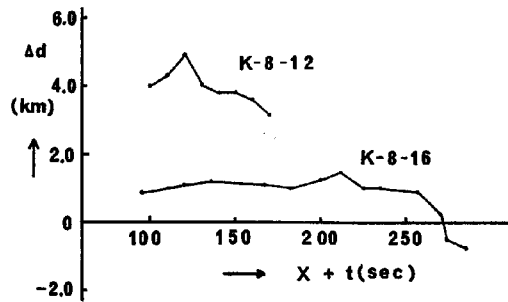


Fig. 8. Absolute deviations Δd in km between this method and radar method. X is the time of rocket launching.

4.2. Measurement of Upper Atmospheric Wind Velocity

The wind velocities and their directions in the upper atmosphere are deduced precisely by this method. Fig. 9 shows the shape and the position of the sodium cloud in the K-8-16 experiment. The cross-sections corresponding to several μ -planes of the cloud obtained from a pair of stations 3 and 4 are also shown in this figure, and the positions of the cross-sections from another pair agreed with these within the error of about 0.1 km. Fig. 10 shows the wind velocities and directions obtained from time sequent projections of the cloud similar to Fig. 9.

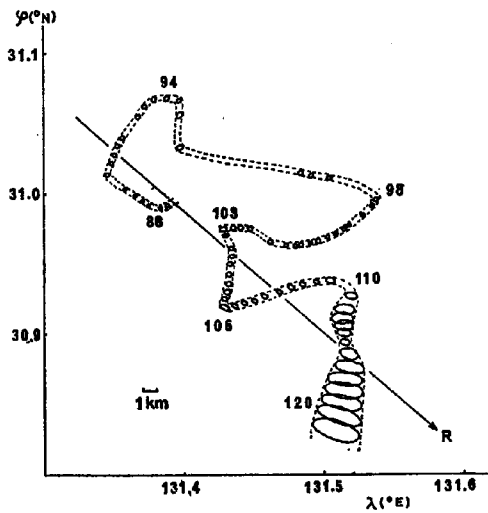


Fig. 9. Level projection of a sodium cloud in the K-8-16 rocket experiment, at $X+197.5$ sec. Cross-sections of cloud deduced from station pair 3-4 are also indicated. The figures beside the projection represent the altitudes of the cloud in km. R shows level projection of rocket trajectory.

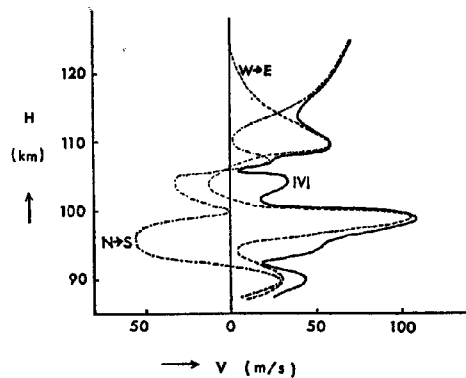


Fig. 10. Wind velocity profile of K-8-16 experiment. The solid curve $|V|$ is the magnitude of the total velocity; the broken line $V(W \rightarrow E)$ is its component from west to east; the chain $V(N \rightarrow S)$ is that from north to south.

4.3. Precision

If the cloud has a clear outline or has a small distance to the observing stations,

the precision of our estimate of the position of the cloud will be improved. Conversely, if the cloud extends and dims or is very distant from the stations, the accuracy of position of the cloud will decline and the error will reach of the order of 1 km.

Thus the accuracy changes with conditions, and we conclude that the highest accuracy gained in this method will be about 0.1 km in position and about several meters per second in velocities (Nakamura *et al.*, 1969), while in the case of K-8-12 experiment, the precision of position is considered as accurate to within 1 km.

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References

- Bečvář, A., "ATLAS COELI II Katalog 1950.0" Nakladatelství Československé Akademie věd Praha (1964).
- Broglio, L., Space Research II, 1125 (1961).
- Justus, C. G., H. D. Edwards and R. N. Fuller, Sci. Rep. Georgia Inst. Tech. **1**, 1 (1964).
- Nakamura, J. and H. Kimura, Proc. 1968 Symp. on Space Obs. in Japan, 115 (in Japanese).
- Nakamura, J. and H. Kimura, Proc. 11th COSPAR Symp. Small Rocket Inst, Tech., 130 (1969).