

Sporadic-E formation by wind shear, comparison between observation and theory

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A comparison is made between observation and theory of simultaneously observed wind shear and electron density in the lower ionosphere. The observation was done by K-8-16 rocket flown from Kagoshima Space Center, Japan. The method of characteristics for time-dependent solution of the equation of continuity, which is a partial differential equation of the first order, was used. The calculation shows a remarkable consistency between the observation and the theory even in details. The result strongly supports the wind-shear theory for sporadic-E formation. The solution suggests that the effective recombination rate is 10^{-8} cm³ sec⁻¹ as found from other ionospheric observations.

1. INTRODUCTION

Simultaneous observation of the electron density and the wind structure in the lower ionosphere was performed by a K-8-16 rocket, launched from Kagoshima Space Center (31°15'N, 131°04'E) at 1914 JST on September 2, 1970. At the launching time the solar depression angle was 8°. A sodium cloud was released between the altitudes of 90 and 125 km in the ascent for the wind measurement (Figure 1). Sodium was released again between 170 and 160 km during the descent. The second release was not successful for wind measurement due to very fast diffusion, but the photographs initially taken involved the rocket and were useful in locating the rocket position precisely. The electron density was measured by both the gyroplasma probe and electron current (or Langmuir) probe (Figures 2a and 2b).

There was a rapid change of wind velocity with height between 95 and 115 km. There is a definite correspondence between this wind profile and the electron density observed; the large peak density occurred at 102 km and 105 km in the ascent and at 101 km and 106 km in the descent. Besides these large peaks there were small peaks at 94 km and 103 km in the descent. Note that a large maximum of electron density ($\sim 10^8$ cm⁻³) at around 98 km

in the ascent was due to the chemical release. The gyroplasma probe, an impedance probe, is capable of finding the absolute value of the electron density, while the electron current probe gives only a relative value requiring calibration regularly as denoted by L CAL in Figure 2a. Unfortunately the gyroplasma probe was damaged during the descent, but good coincidence of the results obtained by the two different probes during the ascent suggests that the electron current probe was correctly calibrated during the descent. In this paper we calculate the electron density profile that is consistent with the wind profile, which we assume to be stationary.

2. CALCULATION OF ELECTRON DENSITY FROM WIND SHEAR

We shall attempt to find the relation between the wind shear and the electron density, both of which were measured by the rocket experiment described in section 1 (Figures 1, 2a, and 2b). The electron density is obtained by integrating the following equation of continuity:

$$\partial n / \partial t = q - \alpha n^2 - (\partial / \partial z)(nV) \quad (1)$$

where

t is time
 n is electron density
 α is recombination rate

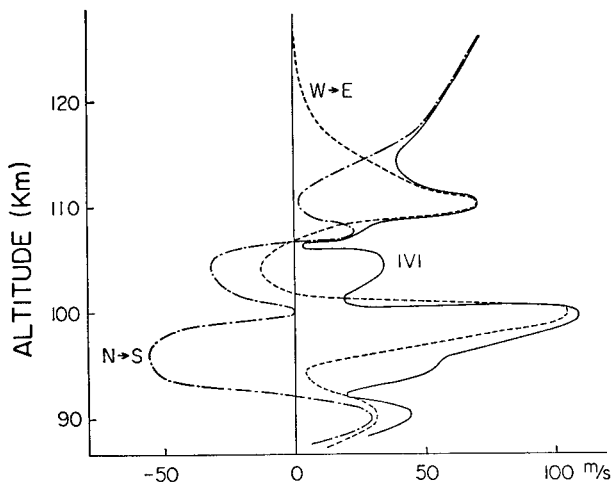


Fig. 1. Observed wind profile. The chain is the southward wind velocity, the broken line is the eastward velocity, and the solid line is the total wind velocity.

q is production rate
 V is vertical ion velocity driven by the wind velocity $W_{x,y}$

The coordinate system is such that the x axis is directed magnetically southward, the y axis magnetically eastward, and the z axis vertically upward. The ion velocity is given by Kato [1966] as

$$V = h(\cos I \sin I)W_x + pW_y \cos I \quad (2)$$

where I is the magnetic dip angle,

$$h = \Omega_i^2 / (\Omega_i^2 + \nu_i^2) \quad (3)$$

$$p = [2\Omega_i \nu_i / (\Omega_i^2 + \nu_i^2)] \sin(I/2)$$

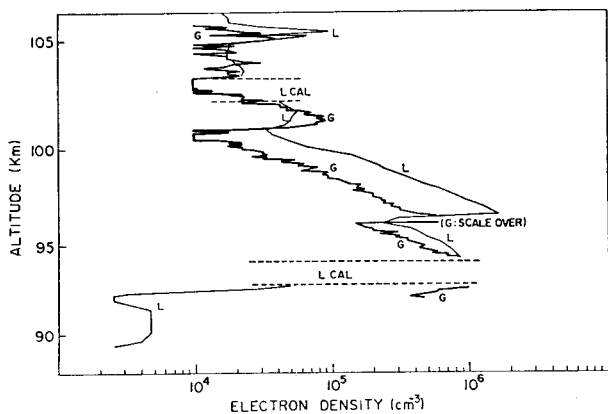


Fig. 2a. Observed electron density distribution measured during ascent. Note that upon chemical release a large increase of electron density occurred around 98 km; G is by the gyroplasma probe and L by the Langmuir probe.

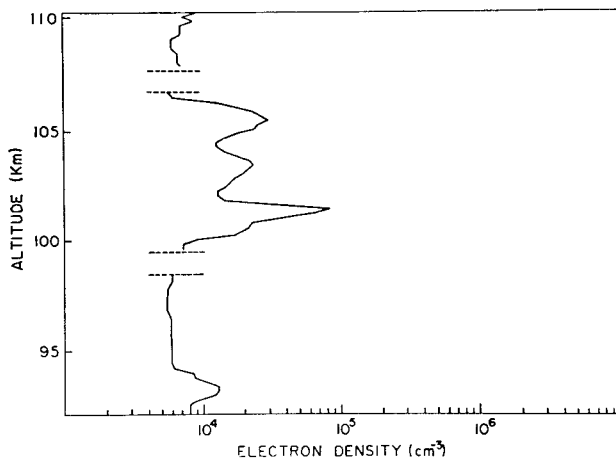


Fig. 2b. Observed electron density distributions measured during descent. The missing parts of the profile marked by the horizontal broken lines show the time when the calibration was done.

Ω_i is ion gyrofrequency, and ν_i is ion collision frequency. Note that $W_{x,y}$ in (2) is obtained from the components in Figure 1 after correction by the magnetic declination of $5^\circ W$. In our calculation $I = 45^\circ$ and

$$\nu_i / \Omega_i = 80 \exp [-(z - 90)/10] \quad (4)$$

where z is measured in km from the ground. Regarding q , we assume

$$q = \alpha n_0^2 \quad (5)$$

where n_0 is the distribution in the absence of V and assumed as

$$n_0 = n_{0_0} \exp [(z - 150)/10] \quad (6)$$

where $n_{0_0} = 1.4 \times 10^5 \text{ cm}^{-3}$. V is shown in Figure 3a.

Equation 1 is a partial differential equation of the first order and is solved by the method of characteristics given by

$$dz/dt = V \quad (7)$$

$$dn/dt = V(dn/dz) = q - \alpha n^2 - n(dV/dz) \quad (8)$$

From (7) we determine the characteristic curve, the trajectory of ionization, as $z = z(z_0, t_0; t)$ where z_0 is a height at $t = t_0$. Along each trajectory, (8) is integrated to give

$$n = n[z(z_0, t_0; t), N_0]$$

where N_0 is n at $z = z_0$ and $t = t_0$, N_0 is n_0 at $t_0 = 0$ given by (6); z_0 , in the calculation, is taken in steps of 1 km from $z_0 = 90$ km to 120 km; and t_0 is taken in

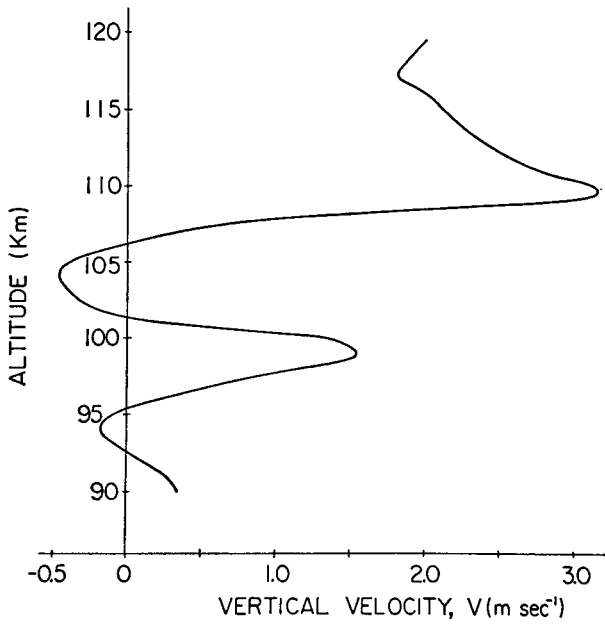


Fig. 3a. V versus height.

steps of 1 min from $t_0 = 0$ to a sufficiently large value to give a quasi-stationary state (Figure 3b). The result is shown in Figures 4a and 4b where $\alpha = 3 \times 10^{-8} \text{ cm}^3 \text{ sec}^{-1}$ and $10^{-8} \text{ cm}^3 \text{ sec}^{-1}$ are adopted and $n(z)$ is illustrated at various times.

It is found that the largest peak appears at $z = 101$ km where $V = 0$ and $dV/dz < 0$. The second largest peak occurs at $z = 93$ km. These results are consistent with a simple theory [Whitehead, 1961]. There are other small peaks, however, at $z = 107$ km, and 118 km. The latter peak almost coincides with the height where V becomes minimum, but the former has no simple corresponding characteristic in V . The characteristic curves shown in Figure 3b show how the ionization piles up with the passage of time at around 93 and 101 km.

3. DISCUSSION

The calculation in section 2 is based on the initial condition that V deduced from the observed $W_{x,y}$ by (2) is suddenly applied on an initial distribution of ionization that is a slowly increasing exponential function with height as given by (6). If $W_{x,y}$ is due to a tidal or gravity wave traveling vertically, such sudden disturbance over the entire range of heights does not really occur. Upon meeting the tidal or the gravity wave, ionization begins to be disturbed gradually. Then, the result for large t in our calculation corresponds to reality. Even for

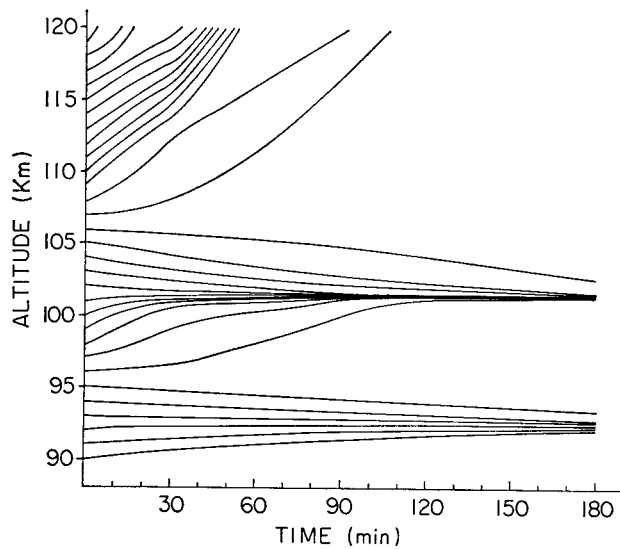


Fig. 3b. Characteristics $z = z(z_0, t_0; t)$ for $t_0 = 0$.

such quasi-stationary state, however, the present mathematical approach is very useful because the boundary condition on n is generally unknown for a stationary state, which makes it difficult to integrate (1) for $\partial n/\partial t = 0$.

In comparing the calculated result (Figures 4a

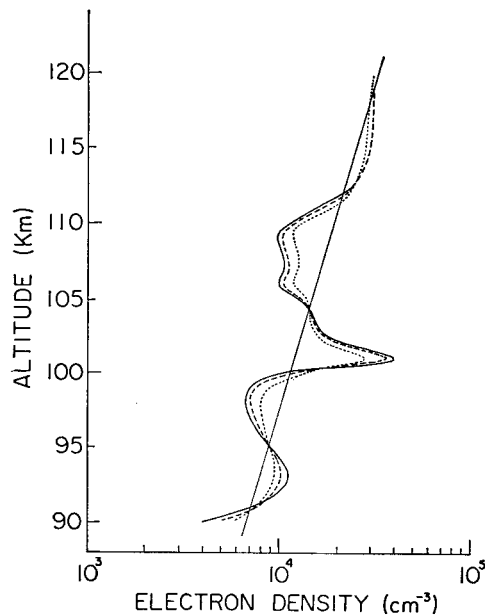


Fig. 4a. Calculated electron density distribution, $n = n(z)$; $\alpha = 3 \times 10^{-8} \text{ cm}^3 \text{ sec}^{-1}$. The dotted curve is $n(z)$ at $t = 15$ min, the broken curve at $t = 30$ min, and the solid curve at $t = 60$ min. The thin straight line is $n_0(z)$.

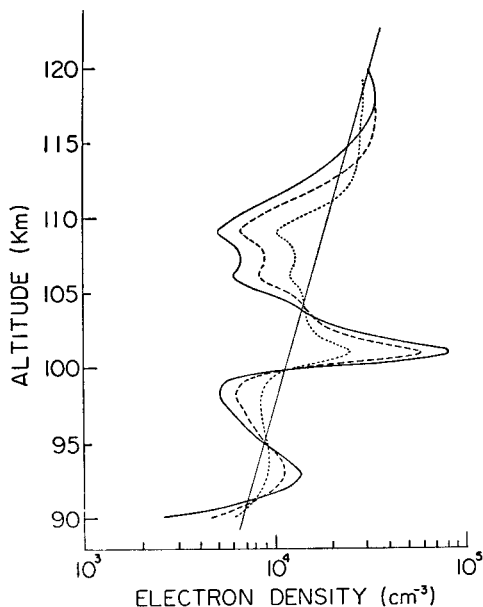


Fig. 4b. Calculated electron density distribution, $n = n(z)$; $\alpha = 10^{-8} \text{ cm}^3 \text{ sec}^{-1}$. The dotted curve is $n(z)$ at $t = 10$ min, the broken curve at $t = 30$ min, and the solid curve at $t = 90$ min. The thin straight line is $n_0(z)$.

and 4b), and the observed (Figure 2b), we see that the calculation for $\alpha = 10^{-8} \text{ cm}^3 \text{ sec}^{-1}$ and $t = 90$ min explains the observation well; 90 min gives an almost stationary state. The largest peak at 101 km has almost the same magnitude as that observed at the same height. A good coincidence also occurs for the second largest peak at 93 km. It is remarkable that a small peak at 107 km in the calculation corresponds to that at 103 km in the observations. The smaller magnitude in the calculation depends on the recombination constant with z . If α decreases with height, the observed magnitude can be better explained. The peak at 116 km in the calculation, does not correspond well with the

peak at 106 km in the observation. Considering, however, that the horizontal distance between the location of the sodium cloud and that where the density is measured is several tens of kilometers, the coincidence is surprisingly good. Another complicating factor is that the smaller the electron density the less accurate the measured value for the dc current probe which measures the intensity of electron current. As noted in the introduction, there was contamination by the sodium cloud in the ascent (Figure 2a).

It is found that the observed peaked electron density profile is best explained by the observed wind shear structure for $\alpha = 10^{-8} \text{ cm}^3 \text{ sec}^{-1}$. A similar small recombination rate has also been found in many other ionospheric phenomena (e.g. Yonezawa, 1968). Although the laboratory experiment seems to support a larger rate of $10^{-7} \text{ cm}^3 \text{ sec}^{-1}$, which presents a 'recombination problem' in all theories of sporadic E , the data are far from conclusive [O'Malley, 1969; Bardsley, 1970]. If there is a large percentage of vibrationally excited molecules in the ionosphere, the effective recombination rate can be small.

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REFERENCES

- Bardsley, J. N. (1970), Temperature dependence of dissociative recombination, *Phys. Rev.*, *A2* (2), 1359-1365.
 Kato, S. (1966), Theoretical basis of the formation of sporadic E due to wind motion in the ionosphere, *Radio Sci.*, *1* (2), 226-234.
 O'Malley, T. F. (1969), Theory of dissociative recombination in vibrationally excited gases, *Phys. Rev.*, *185* (1), 101-104.
 Whitehead, J. D. (1961), The formation of the sporadic E in the temperate zones, *J. Atmos. Terr. Phys.*, *20* (1), 49-58.
 Yonezawa, T. (1968), A consideration of the effective recombination coefficient in the E -region of the ionosphere, *J. Atmos. Terr. Phys.*, *30* (3), 473-478.