

# Generalized Method for Solving Logical Loops in Reliability Analysis

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**Abstract:** A procedure is proposed for obtaining reliability of a system with loop structure(s). The procedure is not a recursive method but a straight forward method, and directly reaches the exact solution of Boolean equation which represents operating states of a system with loop structure(s). The concepts of “support gap” and “Takeover” phenomenon are introduced for the generalized method. A fundamental loop structure is taken up, and analyzed by the proposed procedure. The result shows an important role of loop structure for maintaining the overall system reliability. The procedure is very useful in evaluating engineering systems which have logical loop structure(s), and also useful in effectively designing high reliable systems.

**Keywords:** Logical loop, Reliability analysis, Boolean system, Support gap, Takeover.

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## 1. INTRODUCTION

For a system which has logical loop structure(s), the Boolean relations have to be described with unknown variable(s). The number of unknown variables equals to the number of essential logical loop structures existed in the system. If we try to solve the Boolean equation(s) with unknown variable(s), we encounter infinite circulation of the unknown variable(s). Logical loop was not generally solved in terms of the arithmetic operators of Boolean algebra. Many attempts[1-4] have been proposed. An exact method[5] has been proposed for solving this problem in reliability analysis, but it was effective only in a restricted analysis condition, that is, in the condition that almost all the components are started at the same time.

Many kinds of components in engineering systems require supports of other components for their operation. For example, an internal-combustion engine requires fuel supply for its operation. Therefore, the availability of this kind of components must be evaluated in relation to the supporting components. The concepts of “support gap” and “Takeover” phenomenon are introduced for formulation of generalized method for solving logical loop structure(s).

In the present paper, a generalized procedure is proposed for determining the terms which represent the operating state of loop structure. The procedure is not a recursive method but a straight forward method, and directly reaches the exact solution of Boolean equation which represents operating states of a system with loop structure(s). The generalized procedure is applicable to the condition that components can start at any time in system operational sequence, and each component has multiple chances to be started.

A sample system, which has a fundamental loop structure, is taken up and solved by this procedure with detailed explanations. The result shows an important role of loop structure for maintaining the overall system reliability.

The procedure is very useful in evaluating engineering systems which have logical loop structure(s), and also useful in effectively designing high reliable systems.

## 2. TYPES OF COMPONENTS

If relations between supporting and supported components in a large system produce a closed circuit, a logical loop structure exists in the system. From engineering considerations[6], it is revealed that three different types of components exist in relation to the operating state of loop structure. They are self sustained type, generative type and transmitter type.

### 2.1. Self sustained type component

A self sustained type component (called as "SS-type") is a component which can start and continue its operation without any support by other component. The SS-type component does not require support. Examples of this type component are battery, radio isotopic powered generator, accumulator tank, water tank placed at high position, external electric power source and so on. If a start signal or command is given to a SS-type component, it begins and continues its operation without any support (input) from outside.

## 2.2. Generative type component

A generative type component (called as "G-type") is a component which can produce a driving force for loop operation, in other words, supplies sufficient energy to loop operation. Examples of this type component are engine, electric motor, pump, etc. An engine and a motor produce power for shaft rotation, and a pump produces water flow. But, a G-type component requires support by other component for the production of its output. If there is no support to a G-type component, it cannot start its operation even if the G-type component is in sound state. An engine requires fuel supply, an electric motor requires electricity, and a pump requires both water supply and electricity.

If a sub-system consists of many components including SS-type or G-type, but it requires support from outside of sub-system for its operation, this sub-system can be considered as one G-type (or T-type) component.

## 2.3. Transmitter type component

A transmitter type component (called as "T-type") is a component which cannot produce a driving force for loop operation, in other words, does not produce sufficient energy required for loop operation. Examples of this type component are pipe, wire, an electric transformer, an energy converter, etc. These components just transfer input to output. Pipe and wire transmit exactly the same thing, that is, input and output have same quality, water or electric power or signal. An electric transformer and energy converter modify inputs and produce some different quality of outputs, but they do not give driving force.

A T-type component also requires input from other component for the production of its output. If there is no input to a T-type component, it cannot send out its output even if the T-type component is in sound state.

For an operable engineering system, the connections between components are designed to be always consistent. Proper input is given to a component and its output is adequately connected to other component. For a continuous loop operation under isolated condition, it is essential for G-type component(s) to be existed in the loop structure.

## 3. EXPRESSION OF OPERATING STATE OF COMPONENTS

In the following, irreversible change of component's state is assumed. A set representing the sound state of component A in standby condition at time t is expressed by the notation  $S_A(t)$ . Then, the probability of component A's soundness becomes as follows, for the constant standby failure rate  $\lambda_{AS}$ .

$$\Pr(S_A(t - t_0)) = S_{A0} \cdot \exp(-\lambda_{AS}(t - t_0)), \quad (1)$$

where,  $S_{A0}$  is the probability that component A is in sound state at time  $t_0$  (initial time).

The notation  $\sigma_{A,t}$  is used for a set representing the event, in which starting signal is given to component A at time t. The notation  $D_{A,t}$  is used for a set representing the event, in which component A successfully starts its operation by a given starting signal. Then,  $\Pr(D_{A,t})$  equals demand probability.

A notation  $O_A(t)$  is used for a set representing the operable state of component A after it has been operated during time duration t. For the constant operating failure rate,  $\Pr(O_A(t))$  becomes as follows,

$$\Pr(O_A(t - t_1)) = \exp(-\lambda_{AO}(t - t_1)), \quad (2)$$

where,  $t_1$  is the time in which component A starts its operation.

If component A is a SS-type and a start signal has been given at time  $t_l$ , then the component A's operating state becomes as follows,

$$A_o(t) = S_A(t_l - t_0) \cdot \sigma_{A,t_l} \cdot D_{A,t_l} \cdot O_A(t - t_l), \quad (3)$$

where  $A_o(t)$  is a set representing the state in which component A is in operating state and produces output at time  $t$ . After the start of component A's operation,  $t_0, t_l$  becomes a fixed time and not a variable. Then, the time  $t_0, t_l$  are rewritten by  $\tau_0, \tau_l$ , as they can be easily distinguished from variable  $t$ .

$$A_o(t) = S_A(\tau_l - \tau_0) \cdot \sigma_{A,\tau_l} \cdot D_{A,\tau_l} \cdot O_A(t - \tau_l). \quad (4)$$

If more than one start signals are given to component A, operating state which started by  $i$ -th start signal becomes as follows,

$$A_o^i(t) = \prod_{j=1}^{i-1} \overline{\sigma_{A,j} \cdot D_{A,j}} \cdot S_A(\tau_{A,i} - \tau_0) \cdot \sigma_{A,i} \cdot D_{A,i} \cdot O_A(t - \tau_{A,i}) = \prod_{j=1}^{i-1} \overline{\sigma_{A,j} \cdot D_{A,j}} \cdot A^i(t). \quad (5)$$

Where,  $\sigma_{A,j}$  means  $j$ -th start signal given to component A, and  $\tau_{A,j}$  is the time in which start signal  $\sigma_{A,j}$  is given to A. A notation  $\overline{\sigma_{A,j} \cdot D_{A,j}}$  is the complement of  $\sigma_{A,j} \cdot D_{A,j}$  and represents a set in which component A did not start at time  $\tau_{A,j}$ . It is assumed that  $\sigma_{A,j}$  and  $\sigma_{A,k}$ , or  $D_{A,j}$  and  $D_{A,k}$  are mutually independent.

Notation  $A^i(t)$  is an abbreviated expression of  $S_A(\tau_{A,i} - \tau_0) \cdot \sigma_{A,i} \cdot D_{A,i} \cdot O_A(t - \tau_{A,i})$ , which means that component A is started at time  $t_{A,i}$  (at the time of  $i$ -th starting chance) and continues to operate till  $t$  under the isolated condition. Therefore,  $A^i(t)$  expresses elementally characteristics of component A itself, and does not express the actual operating state of component A.

Equation (5) indicates the situation that once a component becomes in operating state it never start again. In general, operating state of component A at time  $t$  becomes as follows,

$$A_o(t) = \sum_{i=1}^{t_{A,i} \leq t} A_o^i(t). \quad (6)$$

For a G-type or a T-type component, support by other component is required for its operation. Then,

$$B_o^1(t) = A_o(t) \cdot B^1(t) = A_o(t) \cdot S_B(\tau_1 - \tau_0) \cdot \sigma_{B,1} \cdot D_{B,1} \cdot O_B(t - \tau_1), \quad (7)$$

where B is a G-type or a T-type component and is directly supported by a SS-type component A. Equation (7) represents the state in which component B starts at time  $t_l$  and it is the first chance to start.

If more than one start signals are given to components A and B, the operating state of  $k$ -th start of B under the support of  $A_o^i(t)$  becomes as follows,

$$B_o^k(t) = \prod_{j=1}^{i-1} \left\{ A_o^j(\tau_j) + A_o^i(\tau_j) \prod_{\tau_{B,m} \geq \tau_{A,j}}^{\tau_{B,m} < \tau_{A,i}} \overline{\sigma_{B,m} D_{B,m}} \right\} \cdot A_o^i(t) \cdot \prod_{t_{B,l} \geq t_{A,i}}^{k-1} \overline{\sigma_{B,l} D_{B,l}} \cdot B^k(t). \quad (8)$$

The factor in the bracket represents the state in which component B did not start before time  $t_{B,k}$ . There is a relation  $A_o^i(t) \cdot A_o^j(t) = \phi$ ; ( $i \neq j$ ), and then equation (8) becomes equation (9).

$$B_o^k(t) = A_o^i(t) \cdot \prod_{t_{B,l} \geq t_{A,i}}^{k-1} \overline{\sigma_{B,l} D_{B,l}} \cdot B^k(t). \quad (9)$$

Equation (9) is generalized as follows,

$$B_o(t) = \sum_{i=1}^{t_{A,i} \leq t} \left[ A_o^i(t) \cdot \sum_{\substack{t_{B,k} \leq t \\ t_{B,l} \geq t_{A,i}}} \prod_{l=1}^{k-1} \overline{\sigma_{B,l} D_{B,l}} \cdot B^k(t) \right]. \quad (10)$$

If third component C(G- or T-type) is connected after component B, the operating state of component C becomes,

$$C_o(t) = \sum_{i=1}^{t_{A,i} \leq t} A_o^i(t) \cdot \left\{ \sum_{\substack{t_{B,k} \leq t \\ t_{B,l} \geq t_{A,i}}} \prod_{l=1}^{k-1} \overline{\sigma_{B,l} D_{B,l}} \cdot B^k(t) \cdot \left( \sum_{\substack{t_{C,m} \leq t \\ t_{C,n} \geq t_{B,k}}} \prod_{n=1}^{m-1} \overline{\sigma_{C,n} D_{C,n}} \cdot C^m(t) \right) \right\}. \quad (11)$$

In the above equation, it is assumed that the operation of component B is possible regardless of the state of component C.

#### 4. SUPPORT GAP AND "TAKEOVER" PHENOMENON

Now consider the case three components are connected as shown in Fig. 1. Components A and B are SS-type, and component C is a G- or T-type. Start signals to their operations are given at time  $t_a$ ,  $t_b$  and  $t_c$ , respectively.

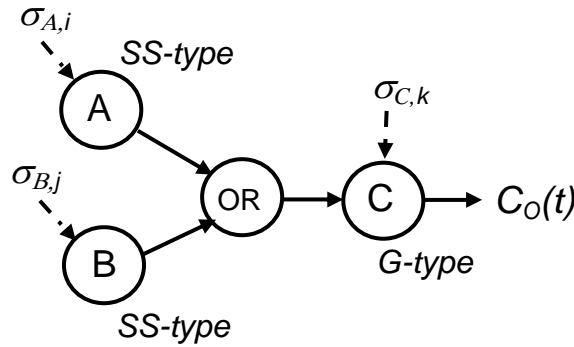


Figure 1. Parallel supports by two components.

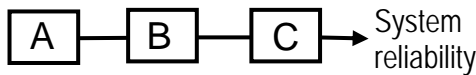


Figure 2. Series connection in RBD.

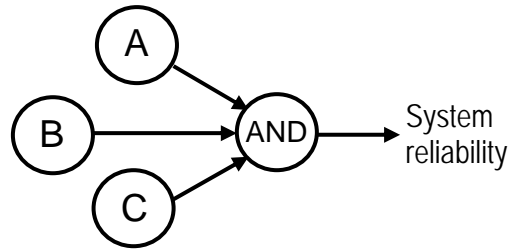


Figure 3. Expression by support relation.

Attention has to be made for the expression of figure 1. The arrows which connect components express the supporting relations. But, in usual reliability block diagram(RBD) the connecting lines between components do not mean the supporting relation. The relation shown in figure 2 is transformed into figure 3 by using the supporting relation (arrow).

For the time sequence of  $t_a < t_b < t_c$  in figure 1, output of component C becomes as follows,

$$C_o(t) = \{A_o(t) + B_o(t)\} \cdot C^1(t) = \{A_o(t) + B_o(t)\} \cdot S_c(\tau_{c,1} - \tau_0) \cdot \sigma_{c,1} \cdot D_{c,1} \cdot O_c(t - \tau_{c,1}). \quad ; \quad t > \tau_{c,1}. \quad (12)$$

As start signals are given successively in the order of component links, the start of downstream component is possible. The link with this condition is called "connecting chain" in this paper.

For a time sequence of  $t_c < t_a < t_b$ , a start signal to component C is given in advance to components A and B's operation. Then, component C never becomes operating state.

$$C_o(t) = \phi. \quad (13)$$

Component A and B do not support the start of component C. There is a "support gap" between them, and the links A-C, B-C do not become connecting chains.

For the time sequence of  $t_a < t_c < t_b$ , component C can start its operation by the support of component A, but not by the component B. There is a "support gap" between B and C, that is, component B does not support the "start" of component C. But, B begins to support the "operation" of component C after time  $t_b$ . The operating state of component C becomes as follows,

$$\begin{aligned}
C_o(t) &= A_o(t) \cdot S_c(\tau_{c,l} - \tau_0) \cdot \sigma_{c,l} \cdot D_{c,l} \cdot O_c(t - \tau_{c,l}) + B_o(t) \cdot A(\tau_b) \cdot S_c(\tau_{c,l} - \tau_0) \cdot \sigma_{c,l} \cdot D_{c,l} \cdot O_c(t - \tau_{c,l}) \\
&= A_o(t) \cdot C(t) + B_o(t) \cdot A(\tau_b) \cdot C(t). \quad ; \quad t > t_b.
\end{aligned} \tag{14}$$

The second term in the equation (14) represents this contribution, and this situation is called as "takeover" phenomenon in this paper. Even if there is a "support gap" between B and C, the link B-C becomes "connecting chain" by the "takeover" phenomenon.

Now consider the case more than one start signals are given to each component for the connecting relation of components shown in figure 1. If component A does not become in operating state before  $t_{B,j}$ , operating state of component C started under the support of  $B_o^j(t)$  becomes as follows,

$$B_o^j(t) \sum_{t_{c,k} \geq t_{B,j}} \cdot \prod_{t_{B,j} \leq t_{c,l}}^{t_{c,l} \leq t_{c,k}} \sigma_{c,l} D_{c,l} \cdot C^k(t) \tag{15}$$

The above equation has the same form to the inside of the bracket of equation (10).

If component A is started before  $B_o^j(t)$ , operating state of component C started under the support of  $B_o^j(t)$  becomes as follows,

$$\prod_{i=1}^{\tau_{A,i} < \tau_{B,j}} \left\{ A_o^i(\tau_{A,i}) + A_o^i(\tau_{A,i}) \prod_{\tau_{c,n} \geq \tau_{A,i}}^{\tau_{c,n} < \tau_{B,j}} \sigma_{c,n} D_{c,n} \right\} \cdot B_o^j(t) \sum_{t_{c,k} \geq t_{B,j}} \cdot \prod_{t_{B,j} \leq t_{c,l}}^{t_{c,l} \leq t_{c,k}} \sigma_{c,l} D_{c,l} \cdot C^k(t). \tag{16}$$

This equation is similar to the equation (8), and terms  $A_o^i(t)$ ,  $B^k(t)$  are replaced by  $B_o^j(t)$ ,  $C^k(t)$ . As the relation  $A_o^i(t) \cdot B_o^j(t) \neq \phi$  exists, a factor of bracket does not disappear.

If operation of component B is started after  $C_o^k(t)$ , "takeover" phenomenon appears and the operating state of component C supported by  $B_o^j(t)$  becomes as follows,

$$B_o^j(t) \left\{ \sum_{i=1}^{t_{A,i} \leq t_{c,k}} A_o^i(\tau_{B,j}) \sum_{t_{A,j} \leq t_{c,k}} \cdot \prod_{t_{A,j} \leq t_{c,l}}^{t_{c,l} \leq t_{c,k}} \sigma_{c,l} D_{c,l} \cdot C^k(t) \right\}. \tag{18}$$

## 5. OPERATE FUNCTION $T \left\{ \begin{matrix} \\ \end{matrix} \right\}_{t \rightarrow \tau}$

As seen in the previous section, "takeover" phenomenon occurs when additional support is supplied to a component which is already in an operating state. Let operating component denote Q and supporting component denote I. A set representing operating state of component Q is  $Q_o(t) = I_o(t)Q(t)$ . If an additional support J is supplied to Q at time  $t_j$ , operating state of Q becomes as shown in the next equation.

$$Q_o(t) = I_o(t) \cdot Q(t) + J_o(t) I_o(\tau_j) \cdot Q(t). \tag{19}$$

The second term is the contribution generated by the additional support.

Now introduce an operate function  $T \left\{ \begin{matrix} \\ \end{matrix} \right\}_{t \rightarrow \tau}$  for the calculation of takeover effects. This function replaces the time variable  $t$  to a fixed time  $\tau$  (the time additional support is supplied) in the supporting part of original operating event.

$$\{\text{Additional operating event}\} = \{\text{Additional support}\} \cdot T \left\{ \begin{matrix} \\ \end{matrix} \right\}_{t \rightarrow \tau} \{\text{Original operating event}\} \tag{20}$$

For example, equation (18) is derived as follows. Operating state of component C started at time  $t_{c,k}$  is,

$$C_o^k(t) = \sum_{i=1}^{t_{A,i} \leq t_{c,k}} A_o^i(t) \prod_{t_{A,j} \leq t_{c,l}}^{t_{c,l} \leq t_{c,k}} \sigma_{c,l} D_{c,l} \cdot C^k(t). \tag{21}$$

An additional support is  $B_o^j(t)$ , which is supplied at time  $t_{B,j} (> t_{c,k})$ . Then, additional operating event becomes,

$$\{\text{Additional operating event}\} = B_o^j(t) \cdot T \left\{ \begin{matrix} \\ \end{matrix} \right\}_{t \rightarrow \tau_{B,j}} \left\{ C_o^k(t) \right\} = B_o^j(t) \cdot \sum_{i=1}^{t_{A,i} \leq t_{c,k}} A_o^i(\tau_{B,j}) \prod_{t_{A,j} \leq t_{c,l}}^{t_{c,l} \leq t_{c,k}} \sigma_{c,l} D_{c,l} \cdot C^k(t). \tag{22}$$

This equation means that operation of component C is supported by component B instead of A, in additional operating event.

## 6. ANALYSIS BY BOOLEAN ALGEBRAIC EQUATION

As a fundamental configuration of loop structure, we take up a system shown in Fig. 4. Components S1 and S2 are SS-type and components A, B and C are G-type components in this case. Time sequence of  $t_{s1} = t_{s2}(=t1) < t_b(=t2) < t_a(=t3) < t_c(=t4)$  is assumed, and start signals  $\sigma_{S1}$ ,  $\sigma_{S2}$ ,  $\sigma_B$ ,  $\sigma_A$  and  $\sigma_C$  are given at the corresponding time.

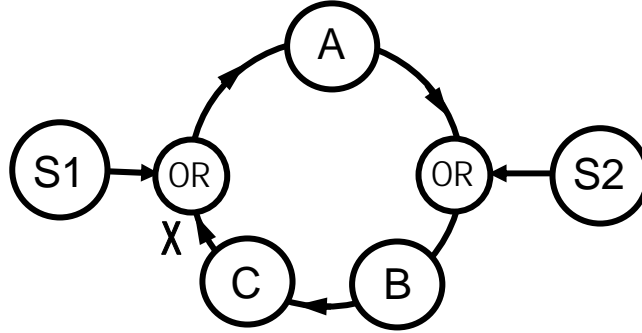


Figure 4. Fundamental loop structure with two SS-type components.

The followings are operating states of components A, B and C after the establishment of loop operation, obtained by step by step calculation procedure by using an operate function  $T \left\{ \begin{matrix} \\ \end{matrix} \right\}$ .

$$\begin{aligned}
 A_o(t) &= S1_o(t)A^1(t) + S1_o(\tau_4)S2_o(\tau_3) \cdot A^1(t)B^1(t)C^1(t), \\
 B_o(t) &= S2_o(t) \cdot B^1(t) + S1_o(t)A^1(t) \cdot S2_o(\tau_3) \cdot B^1(t) + S1_o(\tau_4)S2_o(\tau_3) \cdot A^1(t)B^1(t)C^1(t), \\
 C_o(t) &= S2_o(t) \cdot B^1(t)C^1(t) + S1_o(\tau_4)S2_o(\tau_3) \cdot A^1(t)B^1(t)C^1(t).
 \end{aligned} \tag{23}$$

The system shown in Fig 4. can be expressed by the next Boolean equation.

$$X = \{(S1 + X) \cdot A + S2\} \cdot B \cdot C. \tag{24}$$

The  $X$  is a set representing the state in which output of C exists. Elements  $A, B, C, S1, S2$  are sets representing the state in which corresponding components are in operating states. "Operating state" is rather conceptual at this stage. Concrete expressions are identified after solving the equation (24).

The equation (24) is constructed with the condition that G- or T-type components require supports for their operation. The equation (24) contains all the possible operating states of component C at any given time.

Open the brackets,

$$X = S1 \cdot A \cdot B \cdot C + A \cdot B \cdot C \cdot X + S2 \cdot B \cdot C. \tag{25}$$

Element  $X$  appears in the second term, that is, output  $X$  itself is used as an input. Then the second term represents a loop structure. The relation expressed by equation (25) does not change after repeating the substitution of  $X$  into the right hand side of equation (25). This is the situation that infinite circulation of unknown element appears in the process of solving a Boolean equation with unknown element(s).

But, the equation can be directly solved with the aids of Boolean fundamental theorems. The solution of the general form becomes as follows [6], which is accompanied with indefinite elements  $m_k$ .

$$\begin{aligned}
 X &= f_1(\alpha_1, \dots, \alpha_n)X + \dots + f_k(\alpha_1, \dots, \alpha_n)X + g(\alpha_1, \dots, \alpha_n) \\
 \Rightarrow X &= m_1 \cdot f_1(\alpha_1, \dots, \alpha_n) + \dots + m_k \cdot f_k(\alpha_1, \dots, \alpha_n) + g(\alpha_1, \dots, \alpha_n).
 \end{aligned} \tag{26}$$

Then, the solution of equation (25) becomes,

$$X = S1 \cdot A \cdot B \cdot C + m \cdot A \cdot B \cdot C + S2 \cdot B \cdot C, \tag{27}$$

Substitute the concrete expression of operating states obtained as (23) into each term of equation (27), and rearrange,

$$\begin{aligned}
 S1 \cdot A \cdot B \cdot C &= S1_o(t)S2_o(\tau_3) \cdot A^1(t)B^1(t)C^1(t), \\
 m \cdot A \cdot B \cdot C &= m \cdot S1_o(\tau_4)S2_o(\tau_3) \cdot A^1(t)B^1(t)C^1(t), \\
 S2 \cdot B \cdot C &= S2_o(t) \cdot B^1(t)C^1(t).
 \end{aligned}
 \tag{28}$$

Above results are at the time after the establishment of loop operation.

The first term is operating state produced by non-loop connection from S1 to C, and the third term is also by non-loop connection from S2 to C. The second term is loop operating state, because there are no  $S1_o(t)$  and  $S2_o(t)$  terms. The loop operating state is obtained as  $S1_o(t_4)S2_o(\tau_3) \cdot A^1(t_4)B^1(t_4)C^1(t_4)$  in the equation (23). Therefore, it is revealed that indefinite element  $m$  equals universal set ( $=I$ ). The first term becomes subset of the second term, and disappears from the final result.

The terms indicated by indefinite element  $m$  or  $m_i$  correspond to a loop type connection without SS-type component. Concrete expression for this kind of term is obtained as follows. Select a component started first among the loop. Identify the operating state of this component at the starting time. It can be determined, because a loop connection is not established at that time and the component is supported by a SS-type component outside of the loop structure. Then, successively determine operating state of connected components along the connection, till loop operating state is obtained. In this case, “takeover” phenomenon is used at least one time for the calculation of connecting chains. An indefinite element  $m$  is set to be  $I$ .

## 7. ANALYSIS EXAMPLE OF LOGICAL LOOP STRUCTURE

A sample system is taken up as shown in Fig. 5 and analysis is performed by the proposed method. By this analysis, it becomes clear the procedure for solving a typical loop structure. The system is a general and rather abstract system which has one loop structure with an external support. External support P(SS-type) is connected to component A (G-type; Electric generator is suggested.). Components A and B (G-type; Fuel and fuel pump is suggested.) make a loop structure. X (It suggests electric power supply to users.) is an output of the system and it includes the operating state of loop structure.

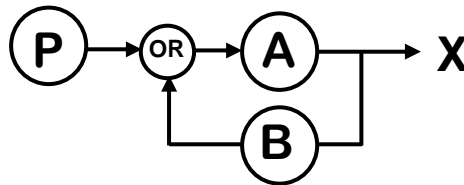


Figure 5. A sample system.

### 7.1. Analysis by step by step procedure

The following time sequence and operational conditions are assumed. External support P starts at the initial time ( $t_0$ ). Component A starts at time  $t_1$  with the support of P, and component B starts at time  $t_2$  with the support of A. Then loop operation established at time  $t_2$ .

Random failures are considered for all the components. In addition to the random failures, stop of component P or B are considered at time  $t_3$ , which is produced by some external cause. Restart of the stopped component is taken at time  $t_4$  with the probability 1.0.

Table 1 shows four analysis conditions which are selected depending on the combination of stopped component and restart action. In the case 2, loop operation is continued under the condition of no external support.

Table 1. Time sequence and operational conditions.

Time	Case 1	Case 2	Case 3	Case 4
$t_0$ :Initial time	P starts	P starts	P starts	P starts
$t_1$	A starts	A starts	A starts	A starts
$t_2$ :Loop operation starts	B starts	B starts	B starts	B starts
$t_3$ :Stop of component P or B	P stops, System stops	P stops, Loop operation continues	B stops	B stops
$t_4$ :Restart of stopped component		P restarts	B restarts Loop operation starts again.	No recovery
$t_5$ :End state	No EP	EP supply	EP(Electric Power) supply	EP supply

Output  $X$  is expressed in the following Boolean equation.

$$X = P \cdot A + A \cdot B \cdot X. \quad (29)$$

The solution of equation (29) becomes,

$$X = P \cdot A + mA \cdot B. \quad (30)$$

At time  $t_1$ , system output ( $X$ ) becomes as follows for all cases,

$$X(t_1) = P_o(t_1) \cdot A_o(t_1) = P_o(t_1) \cdot S_A(\tau_1 - \tau_0) \sigma_{A,t1} D_{A,t1}. \quad (31)$$

At time  $t_2$ , operating state of A is obtained as follows for all cases,

$$A_o(t_2) = P_o(t_2) \cdot A(t_2) = P_o(t_2) \cdot S_A(\tau_1 - \tau_0) \sigma_{A,t1} D_{A,t1} O_A(t - \tau_1).$$

$$B_o(t_2) = A_o(t_2) \cdot B(t_2) = P_o(t_2) \cdot A(t_2) \cdot B(t_2) = P_o(t_2) \cdot S_A(\tau_1 - \tau_0) \sigma_{A,t1} D_{A,t1} O_A(t - \tau_1) \cdot S_B(\tau_2 - \tau_0) \sigma_{B,t2} D_{B,t2}.$$

$$\text{Additional Operating Event of A} = B_o(t_2) \cdot T_{t_2 \rightarrow \tau_2} \{A_o(t_2)\}$$

$$= P_o(\tau_2) \cdot S_A(\tau_1 - \tau_0) \sigma_{A,t1} D_{A,t1} O_A(t_2 - \tau_1) \cdot S_B(\tau_2 - \tau_0) \sigma_{B,t2} D_{B,t2} \cdot P_o(\tau_2) \cdot S_A(\tau_1 - \tau_0) \sigma_{A,t1} D_{A,t1} O_A(t_2 - \tau_1)$$

$$= P_o(\tau_2) \cdot S_A(\tau_1 - \tau_0) \sigma_{A,t1} D_{A,t1} O_A(t_2 - \tau_1) \cdot S_B(\tau_2 - \tau_0) \sigma_{B,t2} D_{B,t2} = P_o(\tau_2) \cdot A(t_2) \cdot B(t_2) \quad (32)$$

$$\therefore A_o(t_2) = P_o(t_2) \cdot A(t_2) + P_o(\tau_2) \cdot A(t_2) \cdot B(t_2) \quad (33)$$

At time  $t_3$ , operating state of A ( $=X(t_3)$ ) becomes as follows for cases 2, 3 and 4.

$$A_o(t_3) = P_o(\tau_2) \cdot A(t_3) \cdot B(t_3); \text{ for case 2, } A_o(t_3) = P_o(t_3) \cdot A(t_3); \text{ for cases 3 and 4.} \quad (34)$$

At time  $t_4$ , stopped component is restarted. Operating state of A ( $=X(t_4)$ ) is calculated as follows by considering takeover phenomenon.

For case 2.

$$\text{Additional Operating Event of A} = P'_o(t_4) \cdot T_{t_4 \rightarrow \tau_4} \{A_o(t_4)\} = P'_o(t_4) \cdot P_o(\tau_2) \cdot A(t_4) \cdot B(\tau_4)$$

$$= P_o(\tau_2) \cdot B(\tau_4) \cdot P'_o(t_4) \cdot A(t_4) \quad (35)$$

$$\therefore A_o(t_4) = P_o(\tau_2) \cdot B(\tau_4) \cdot P'_o(t_4) \cdot A(t_4) + P_o(\tau_2) \cdot A(t_4) \cdot B(t_4), \quad (36)$$

where  $P'_o(t_4)$  means the event in which external support is newly started at time  $t_4$ .  $P'_o(t_4)$  is independent event of  $P_o(\tau_2)$ .

For, case 3.

$$B'_o(t_4) = A_o(t_4) \cdot B'(t_4) = P_o(t_4) \cdot A(t_4) \cdot B'(t_4) = P_o(t_4) \cdot S_A(\tau_1 - \tau_0) \sigma_{A,t1} D_{A,t1} O_A(t_4 - \tau_1) \cdot S'_B(\tau_4 - \tau_4) \sigma_{B,t4} D'_{B,t4}.$$

$$\text{Additional Operating Event of A} = B'_o(t_4) \cdot T_{t_4 \rightarrow \tau_4} \{A_o(t_4)\} = P_o(\tau_4) \cdot A(t_4) \cdot B'(t_4) \cdot P_o(\tau_4) \cdot A(t_4)$$

$$= P_o(\tau_4) \cdot A(t_4) \cdot B'(t_4) \quad (37)$$



$$\therefore A_o(t_4) = P_o(t_4) \cdot A(t_4) + P_o(\tau_4) \cdot A(t_4) \cdot B'(t_4), \quad (38)$$

Therefore, system output  $X(t)$  ( $=A_o(t)$ ) at time  $t_5$  becomes as follows.

$$\begin{aligned} \text{Case 1; } & X(t_5) = \phi, \\ \text{Case 2; } & X(t_5) = P_o(\tau_2) \cdot B(\tau_4) \cdot P'_o(t_5) \cdot A(t_5) + P_o(\tau_2) \cdot A(t_5) \cdot B(t_5), \\ \text{Case 3; } & X(t_5) = P_o(t_5) \cdot A(t_5) + P_o(\tau_4) \cdot A(t_5) \cdot B'(t_5), \\ \text{Case 4; } & X(t_5) = P_o(t_5) \cdot A(t_5). \end{aligned} \quad (39)$$

## 7.2. Analysis by GO-FLOW model

In the previous section, time dependent reliability of a system with loop structure is expressed by Boolean equations without unknown elements as shown in equation (39). Then, the numerical calculations can be easily performed by the GO-FLOW methodology[7]. Take over phenomena, which appear as terms of  $P_o(\tau_2)$ ,  $P_o(\tau_4)$  and  $B(\tau_4)$ , are well modeled by "phased mission operator" in GO-FLOW charts.

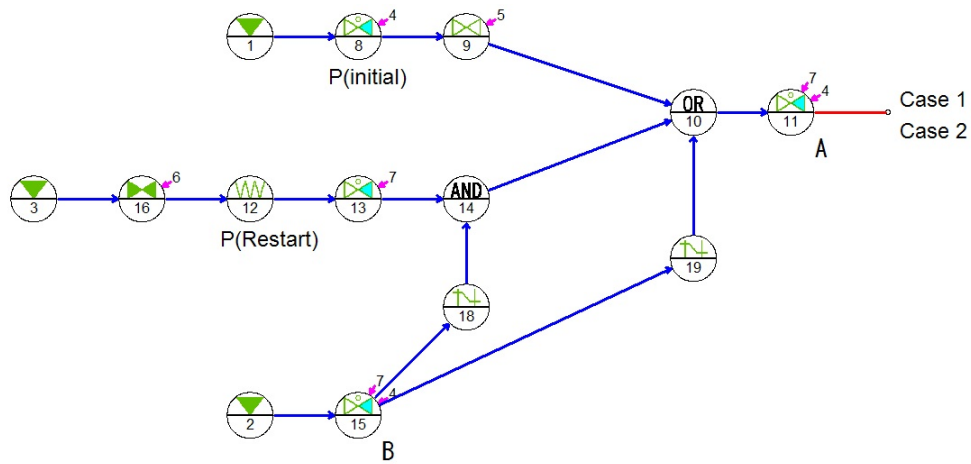


Figure 6. GO-FLOW model for cases 1 and 2.

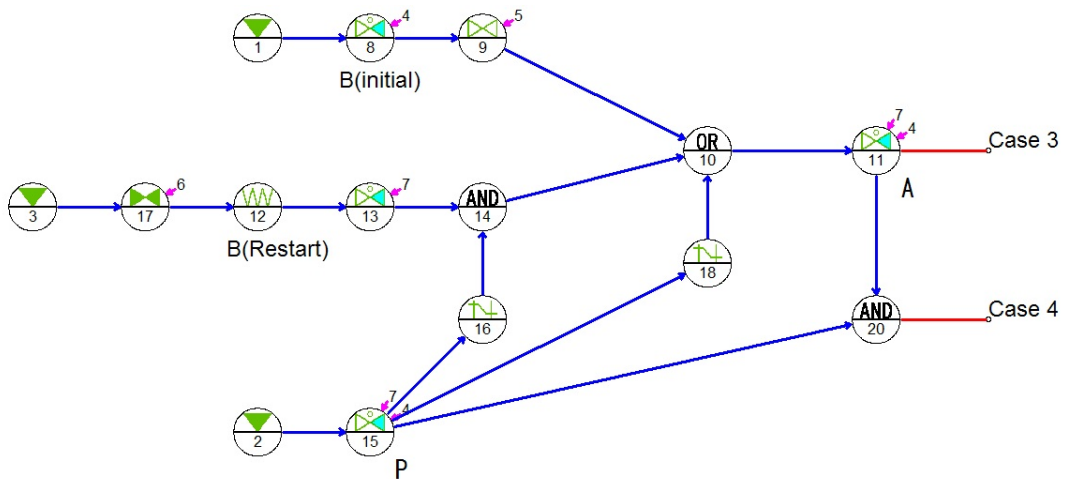


Figure 7. GO-FLOW model for cases 3 and 4.

Analyses are performed with the following failure data. Operating failure rate ( $\lambda_{PO}$ ) of component P equals  $2 \times 10^{-3}/h$ , and  $\lambda_{AO} = \lambda_{BO} = 1 \times 10^{-3}/h$ ,  $t_0 = t_1 = t_2 = 0h$ ,  $t_3 = 20h$ ,  $t_4 = 30h$ ,  $t_5 = 50h$ .

Analysis results are shown in figure 8. The results indicate that high reliability is maintained with the loop operation even after the stop of external supports (case 2), compared with the other cases. The loop structure in this system plays an important role for maintaining the overall system reliability.

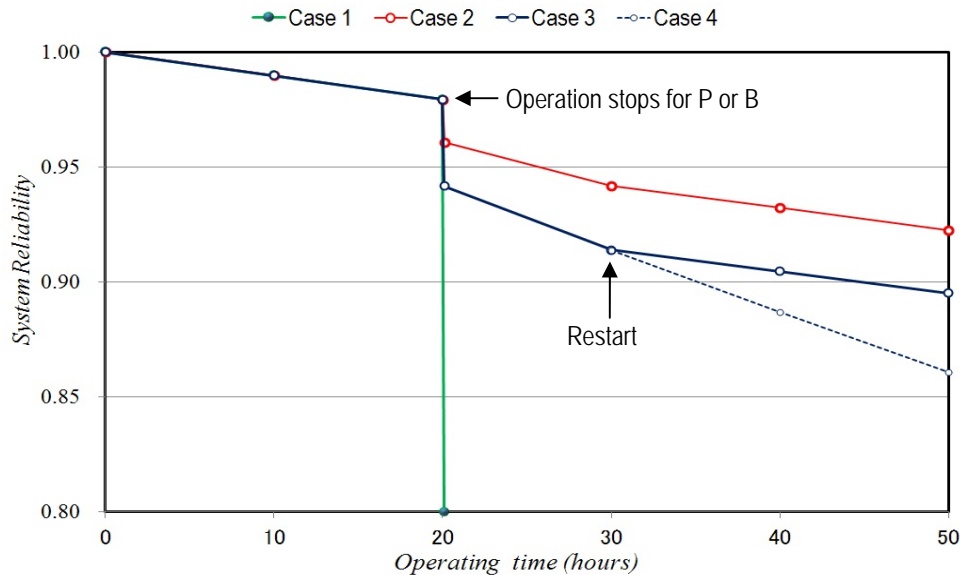


Figure 8. Reliability of a sample system with loop structure.

## 8. CONCLUSIONS

In the present paper, components were classified into three types, SS-type (self sustained type), G-type (generative type) and T-type (transmitter type) based on the engineering considerations about their function.

Expressions were given for operating states of components supported by other components as well as for SS-type component. Concepts of “support gap” and “takeover” phenomenon have been introduced for the explanation of supporting characteristics between components. This takeover phenomenon is essential for the establishment of loop operation. For the calculation of takeover phenomenon, an operate function  $T_{t \rightarrow \tau} \{ \}$  has been introduced.

A procedure by using Boolean equation has been presented for analysis of loop structure(s). The procedure is a straight forward method, and directly reaches the exact solution of Boolean equation which represents operating states of a system with loop structure(s).

A sample system, which has a fundamental loop structure, is taken up and analyzed by step by step procedure. Detailed explanations are given for this analysis process. Numerical calculations have been performed by the GO-FLOW methodology, which can effectively model the takeover phenomenon.

The procedure presented in this paper is very useful in evaluating engineering systems which have logical loop structure(s), and also useful in effectively designing high reliable systems.

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